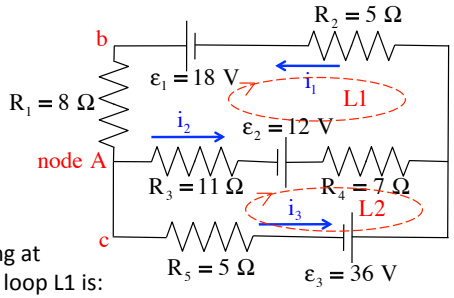
Problem 28.32

This a bizarre problem in the sense that it makes you step through all the math, literally by the numbers. I've dumped everything we need on the sketch to the right!!



a.) For the circuit shown (and starting at position "b"), the Loop Equation for loop L1 is:

$$\text{L1}: \quad -\varepsilon_1 + i_1R_2 + i_2R_4 - \varepsilon_2 + i_2R_3 + i_1R_1 = 0$$
$$-18 + 5i_1 + 7i_2 - 12 + 11i_2 + 8i_1 = 0$$
$$\Rightarrow \quad 13i_1 + 18i_2 = 30$$

b.) For the circuit shown (starting at "c"), the Loop Equation for loop L2 is:

$$\text{L2}: \quad -i_2R_3 + \varepsilon_2 - i_2R_4 - \varepsilon_3 + i_3R_5 = 0$$
$$-11i_2 + 12 - 7i_2 - 36 + 5i_3 = 0$$
$$\Rightarrow \quad -18i_2 + 5i_3 = 24$$

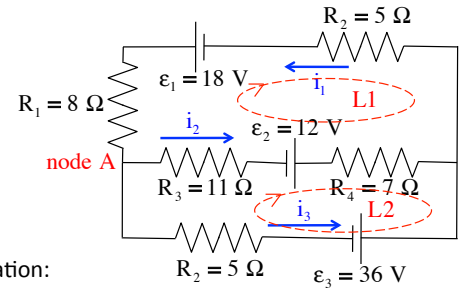
c.) For the circuit shown, the Node Equation for node "A" is:

$$i_1 = i_2 + i_3$$

d.) Solve Node Equation for i_3:

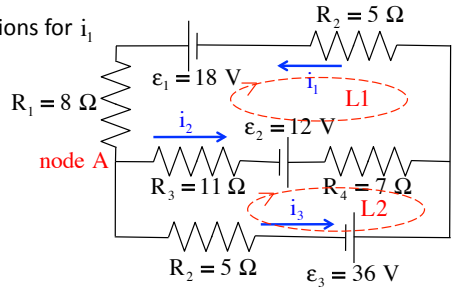
$$i_1 = i_2 + i_3$$
$$\Rightarrow \quad i_3 = i_1 - i_2$$

e.) Substituting i_3 into Loop 2 equation:



$$-18i_2 + 5i_3 = 24$$
$$\Rightarrow \quad -18i_2 + 5(i_1 - i_2) = 24$$
$$\Rightarrow \quad 5i_1 - 23i_2 = 24$$
$$\Rightarrow \quad i_1 = \frac{23i_2 + 24}{5}$$
$$\Rightarrow \quad i_1 = 4.6i_2 + 4.8$$

f.) Solving the modified loop equations for i_1 and i_2, we get:



Using $i_1 = 4.6i_2 + 4.8$:

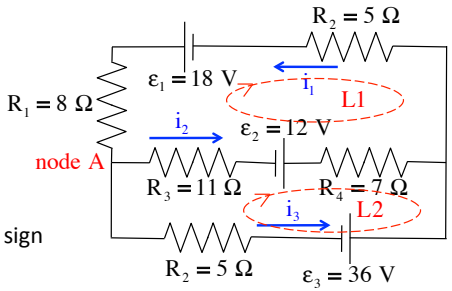
$$13 \quad i_1 \quad + 18i_2 = 30$$
$$13(4.6i_2 + 4.8) + 18i_2 = 30$$
$$\Rightarrow \quad 59.8i_2 + 62.4 + 18i_2 = 30$$
$$\Rightarrow \quad 77.8i_2 + 62.4 = 30$$
$$\Rightarrow \quad 77.8i_2 = -32.4$$
$$\Rightarrow \quad i_2 = -.416\ \text{A}$$

This means:

$$13i_1 + 18 \quad i_2 \quad = 30$$
$$13i_1 + 18(-.416\ \text{A}) = 30$$
$$\Rightarrow \quad i_1 = 2.88\ \text{A}$$

g.) Lastly:

$$i_1 = i_2 + i_3$$
$$\Rightarrow \quad (2.88\ \text{A}) = (-.416\ \text{A}) + i_3$$
$$\Rightarrow \quad i_3 = (3.3\ \text{A})$$



h.) The significance of the negative sign in i_2:

It is not uncommon to not know the actual direction of current in a complicated circuit, so to do a problem like this we have to guess at the current direction. This is not a problem, though, as the math will take care of you. How so? If, given the numbers in the problem, you have assumed the wrong current direction, you will simply find yourself with a negative sign in the offending current's numerical answer. In this case, the current $i_2 = -.416\ \text{A}$ is telling you that the current's magnitude is .416 amps and its direction is not to the right as assumed but, in fact, is to the left. Ain't math wonderful?